Math 3307 Chapter 5 Video Script and Chapter 5 Popper

Now we'll leave descriptive statistics for a bit and study Probability for a bit; this is the lead-in to inferential statistics

5.1 What is Probability?

A probability is a number between 0 and 1 that characterizes the likelihood that something will happen. 0 means it will never happen, 1 means it will definitely happen and proper fractions that are in between these two numbers giving other likelihoods. Sometimes we convert to decimals or percents to describe probability.

We use probabilities to make decisions. If the probability of rain is 80% (4/5) then we'll choose to carry an umbrella. If its 20% (1/5) we won't.

$0 \leq P(x) \leq 1$

5.2 Outcomes and Events

Vocabulary! is needed to discuss probability. OYO and Essay 1 next page

Experiment

Outcomes

Sample Space

Event (subset of sample space)

Probability: $\dfrac{n \text{ favorable outcomes}}{n \text{ total possible}}$ Chapter 6 especially

See the CAUTION on page 123 at the bottom of the page. Resist the urge to reduce fractions! The denominator shouldn't be reduced…it means something as it stands: The total number of possible events in the Universe of the experiment.

Law of Large Numbers

Notation: *P(event)*

Mutually Exclusive Events:

Chapter 5 Essay 1

Write a brief essay tying these vocabulary words into one: Experiment, Outcomes, Sample Space, Event, Probabilty formula in your OWN words.

ONE PAGE, please. I won't grade more than the first page.

Probability Problem 1 - together

Vincent and Joey are playing a game where they each display their right hand with a number of fingers showing simultaneously. (1, 2, 3, or 4 fingers). Try playing this with a family member…count to 3 and then flash your fingers at each other

What are the outcomes in the sample space? Ordered pairs (V, J)

(1,1) (1,2) (1,3) (1,4)

(2,1) (2,2) (2,3) (2,4)

(3,1) (3,2) (3,3) (3,4)

(4,1) (4,2) (4,3) (4,4) event {same # fingers} 4/16 $\frac{n \text{ event}}{n \text{ universe}}$

What is the probability that they each display the SAME number of fingers? Note that 16 in the denominator…if we reduced it some information would be lost!

1/4 reduced … not the "n" we want

Chapter 6 Popper Question 1

From the preceding problem, the probability that Joey shows 4 fingers is:

A. 4/16

B. 4

C. 5/16

D. 0

5.3 Basic Probability Rules (6 rules, more to come later; 4 here)

Rule Number 1

Probabilities are positive and are between 0 and 1.

$0 \leq P(X) \leq 1$

Rule Number 2

The sum of the probabilities of all outcomes is 1. (100%)

Round on the last decimal or stick to fractions.

Rule Number 3

The probability of an event is one minus the probability of its complement.

Note that this text uses a superscripted bar for complement.

$P(A) = 1 - P(\overline{A})$

$P(A) + P(\overline{A}) = 1$

Definition Complement: Everything in the Universe but this event. A^c

Chapter 5 Popper Question 2

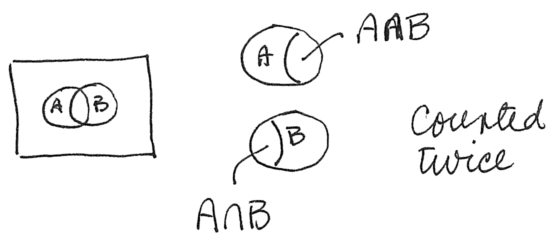
Given the vowels {a, e, i, o, u} and my name: Ms. Leigh. E and I show up in my name. L = {e, i} . L complement is {a, o, u} in the Universe of vowels.

A. True

B. False

Rule Number 4

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$



Chapter 5 Essay 2

State this equation in your own words, filling in for every symbol. For example "P(A)" is "the probability of event A".

Showing this in a VENN diagram for 2 events:

$P(A) + P(B) - P(A \cap B) = P(A \cup B)$

Also note:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) + P(A \cap B) = P(A) + P(B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

Probability Problem 2 together

Given the following probabilities for a number on a LOADED die:

Event	Probability	Common Denominator
1	1/3	4/12
2	1/4	3/12
3	1/12	1/12
4	1/12	1/12
5	1/6	2/12
6	1/12	1/12

Why is it necessary that all the Probabilities add up to one? Do these?

so every thing is accounted for. yes

Find the probabilities of the following events:

A. the number observed is a multiple of 3 $1 \cdot 3^{\checkmark}$ $2 \cdot 3^{\checkmark}$ $3 \cdot 3^{\times}$

$$P(3) + P(6) - P(\underbrace{3 \cap 6}_{0}) = 1/12 + 1/12 = 2/12$$

B. the number is even $\{2, 4, 6\}$ all disjoint! $P(\cap) = 0$

$$\frac{3}{12} + \frac{1}{12} + \frac{1}{12} = \frac{5}{12}$$

C. the number is an even multiple of 3 $\{6\}$

$$P(6) = 1/12$$

Chapter 6 Popper Question 3

Given the table on the preceding page, what is the probability that the number is a prime? Note that 1 is NOT a prime!

A. 1/12

B. 25%

C. 6/12

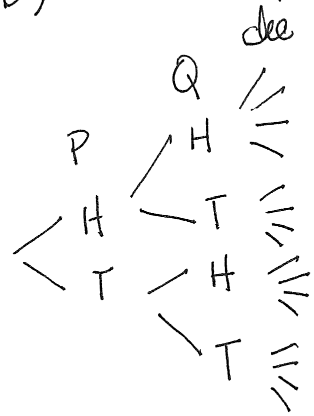
D. 5/12

Probability Problem 3

Use a tree diagram to analyze the following experiment:

You toss a penny, a quarter, and a tetrahedral die (ie 4 sides!). You record the observable face in that order. Fair objects in play!

(P,Q,D)



C.

HH1	HT1	TH1	TT1	
HH2	HT2	TH2	TT2	
HH3	HT3	TH3	TT3	B.
HH4	HT4	TH4	TT4	

A. what is the sample space?

B. what is the probability that a 3 is on the face of the die 4/16

C. what is the probability that 2 heads appear on the coins? 4/16

Probability Problem 4 half together and half homework

Suppose the following probabilities are assigned to these events

P({e1}) = 0.3   P({e2}) = P({e3}) and p({e4}) = 4P({e2})

There are only 4 events in the sample space.

A. find the probabilities assigned to each individual event TOGETHER

ss {e1, e2, e3, e4}

$P(e1) = .3$

$P(e2) = x$

$P(e3) = x$

$P(e4) = 4x$

$6x + .3 = 1$

$6x = .7$

$x = \frac{7}{10} \cdot \frac{1}{6} = \frac{7}{60}$

$4x = \frac{28}{60}$

$0 \leq P(x) \leq 1$

↑ universe

$.3 = \frac{3}{10} \cdot \frac{6}{6} = \frac{18}{60}$

Check

$\frac{18}{60} + \frac{7}{60} + \frac{7}{60} + \frac{28}{60} = \frac{60}{60}$ ✓

✓

Chapter 5 Homework Problem 1, the continuation

B. Find P(A) where A = {e1, e3, e4}

Probability from charts and tables TOGETHER

This is a table of data in thousands for twin births in the USA from

1995 – 1997

The columns are the type of delivery and the rows are the level of care

	C section	Preterm natural	Term	Totals
Intensive	18	15	28	61
Adequate	46	43	65	154
Poor	12	13	38	63
Totals	76	71	131	278

What does each entry in the table mean? Union or Intersection?

$P(\text{Intensive} \cap \text{Preterm N}) = \frac{15}{278}$

disjoint intersections

A. What % of these moms did not receive adequate care during their pregnancies? Note that intensive is also not adequate.

$$P(I) + P(P) = \frac{61+63}{278} = \frac{124}{278} \approx 45\%$$

B. "Given that a mother received poor care," what was the probability that the births were preterm? Remember this in Conditional Prob, next!

changes the sample space we are looking at only the 63 w/. poor care

$$\frac{13}{63} \leftarrow$$

C. Create a appropriate graph comparing the outcomes of these pregnancies by level of medical care the mothers received. What does this graph reveal about medical care for pregnant people carrying twins?

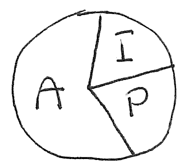
I $\frac{61}{278}$ 22%

A $\frac{154}{278}$ 55%

P $\frac{63}{278}$ 23%

100-77%

$\frac{63}{278}$ .2266.....



for example

Chapter 6 Popper Question 4

Using the above table:

The probability of term intersect intensive is 28/278.

A True

B False

Chapter 5 Homework problem 2, two pages

An experiment consists of selecting a random one of the 100 points in the x-y plane with whole number coordinates from 0 – 9 in x and in y. Hint: sketch this array!

Compute the probability for each of the following events.

A The point lies on the line $y = x$.

B The point lies on the line $y = 2x$.

C The point lies on the circle $x^2 + y^2 = 25$.

D The point lies within but not on the circle $x^2 + y^2 = 25$.

E The point lies within or on the circle $x^2 + y^2 = 25$.

F The point lies outside the circle $x^2 + y^2 = 25$.

G The point lies either on the circle $x^2 + y^2 = 25$ or on the graph of $y = |x - 3|$.

H The point lies on the circle $x^2 + y^2 = 25$ or on the line $y = x$.

I The point lies on the x-axis or the line $y = x$.

J. The sum of the x and y-coordinates of the point is 18.

K The sum of the x and y-coordinates of the point is 21.

L The y-coordinate of the point is greater than the square of the x-coordinate.

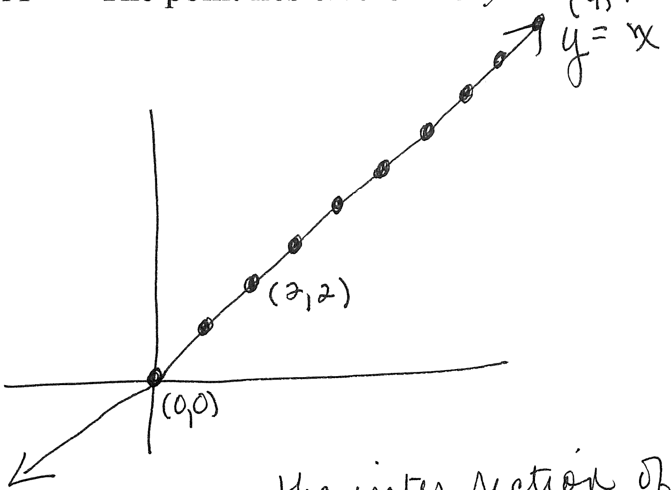
There is some graph paper attached. Feel free to use your own though.

Let's do A together on the next page:

An experiment consists of selecting a random one of the 100 points in the x-y plane with whole number coordinates from 0 – 9 in x and in y. Hint: sketch this array!

Compute the probability for each of the following events.

A The point lies on the line $y = x$. (9,9)



10 dots out of 100 dots

10%

the intersection of the array and the line is 10 dots

Put your array here:

END VIDEO ONE

5.4 Conditional Probability and Independence

Conditions impose restraints on the Sample Space. The probability of A GIVEN that B happened is different than the probability of A alone.

Probability Rule #5: Conditional Probabiltiy

$$(P(A|B) = \frac{P(A \cap B)}{P(B)})$$

Note the change in the Denominator! Just like earlier problem "given poor care"

There are a couple of worked out problems in this section. Be sure to study them closely OYO.

Chapter 6 Essay 3

Compare and contrast the above formula with the original probability formula from page 1:

Probability of a favorable outcome is $\frac{n\ favorable\ outcomes}{n\ total\ possible}$

What has changed?

Now let's discuss **independence** – It's an important concept. Page 137. Let's look at the formula you use if you KNOW events A and B are independent:

$P(A \cap B) = P(A) \cdot P(B)$ if you <u>know</u> independent

$$P(A|B) = P(A)$$

This is quite different from above. Note that you must be able to argue successfully that the events are independent BEFORE you get to use this simpler formula!

Also, not in the book: If two events are independent $P(A \cap B) = P(A) \cdot P(B)$. In general if SMALL samples are taken from a LARGE group without replacement, then you may assume independence.

CP Problem 1 – a continuation

Vincent and Joey are playing a game where they each display their right hand with a number of fingers showing simultaneously. (1, 2, 3, or 4 fingers).

What is the probability that they both display 2 fingers given that they display the same number of fingers?

(1,1) (1,2) (1,3) (1,4)

(2,1) (2,2) (2,3) (2,4)

(3,1) (3,2) (3,3) (3,4)

(4,1) (4,2) (4,3) (4,4)

$\frac{1}{4}$ note diff w/. $\frac{4}{12}$

Let's discuss the difference between now and the first time we worked this!

CP Problem 2 TOGETHER

The probability that the stock market goes up on a Monday is 0.6 and the probability that it goes up on a Tuesday given that it went up on Monday is 0.3. Find the probability that the market goes up both days.

$P(\uparrow M) = .6$ 60%

$P(\uparrow T / \uparrow M) = .3$ 30%

$P(\uparrow M \cap \uparrow T)$

want

$$P(\uparrow T \mid \uparrow M) = \frac{P(\uparrow T \cap \uparrow M)}{P(\uparrow M)}$$

$$.3 = \frac{\text{want}}{.6}$$

$$P(\uparrow T \cap \uparrow M) = .18$$

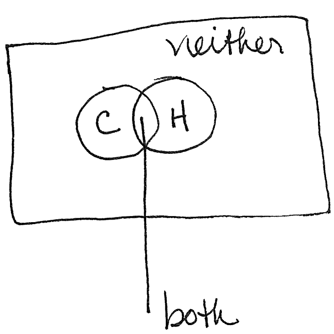
CP Problem 3 – CHAPTER 6 homework problem 3

A cosmetics sales person makes house calls to different neighborhoods. Past experience has shown that the probability that a female resident is home when she calls in the evening is 0.7. Given that the lady of the house is home, the probability of a sale is 0.3. Find the probability that she's home and she buys a product.

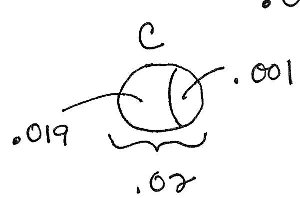
Chapter 5 TOGETHER

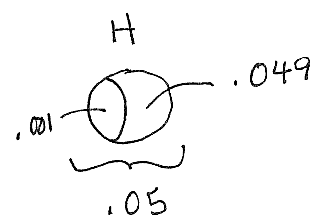
It is known from old medical records that the probability that a person has cancer is 0.02 and the probability of having heart disease is 0.05. Assume that the two ailments are independent events. What is the probability of having both? Neither? One or the other but NOT both?

C = .02   H = .05



P(C∩H) given independence is P(C)·P(H) = .02 × .05 = .001





P(C∪H) = .02 + .05 − .001 = .069

P(neither)

1 − .069 = .9931

P(one or the other not both) = .019 + .049 = .068

5.5 Multiplication Rules

Probability Rule #6   a version of conditional probality

$P(B) \cdot P(A|B) = P(A \cap B)$

This is a version of Rule #5

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Multiply both sides by the probability of event B.

Chapter 6 Popper Question 5

The above formula says that the probability of event B times the probability of event A given event be is:

A the probability of event A union event B

B the probability of event A intersect event B

And recall:

If two events are independent $P(A \cap B) = P(A) \cdot P(B)$

MR Problem 1 - TOGETHER

In a survey of 10,000 African Americans, it was determined that 27 had sickle cell anemia.

Suppose we randomly select one person from the sample. What is the probability that this person has sickle cell anemia?

$$\frac{27}{10,000}$$

If two individuals are selected randomly what is the probability that both have it?

Hint: $P(B)\cdot P(A|B)=P(A\cap B)$ 10,000 is large ... assume indep

$$\frac{27}{10000}\cdot\frac{26}{999}$$

↑

Assuming independence of selection, what is the probability of randomly selecting two people from the sample who have it.

Hint: If two events are independent $P(A\cap B)=P(A)\cdot P(B)$

PIE Problem – Enrichment - TOGETHER

Past attendance records show that the probability that the chair of the board attends the annual sales meeting is 0.65, that the president of the company attends is 0.9, and that they both attend is 0.6. Would you say that they are acting independently? Show the basis of your reasoning.

P(chair) = .65 P(president) = .9 P(both) = .6

P(C) · P(P) = .621

prob. not.

Chapter 5 Homework Problem 4

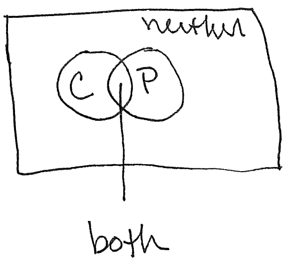
On a TV quiz show a contestant is asked to pick an integer at random from the first 100 natural numbers. Unbeknowst to the contestant, the rule for winning is to pick a number that is divisible by 9 or 12. If a winning number is picked, the contestant will win a trip to the Bahamas.

What is the probability that the contestant will win?

Prob Summary Problem - TOGETHER

The town of Alvin has 2 ambulance services: the city service and a private company. In an emergency, the probability that the city service responds is 0.5, the probability that the private company responds is 0.6, and the probability that either but not both responds is 0.7. Find the probability that both services will respond to an emergency…i.e. find the intersection probability! And the Probability of neither responding?

Let's do a Venn diagram



$P(C) = .5$ $\quad P(P) = .6$

either but not both $.7$

$P(C \cup P) - P(C \cap P)$

$P(C \cup P) = P(C) + P(P) - P(C \cap P)$ ⊖ subtr $\cap$ b.s.

$$\underbrace{P(C \cup P) - P(C \cap P)}_{.7} = P(C) + P(P) - 2P(C \cap P)$$

$$= .5 + .6 - 2P(C \cap P)$$

$$.7 = 1.1 - 2P(C \cap P)$$

$$-.4 = -2P(C \cap P)$$

$$.2 = P(C \cap P)$$

Neither! is $\overline{(P \cup C)}$ $\qquad P(P \cup C) = .5 + .6 - .2 = .9$

$(P \cup C)^c = 1 - .9 = .1$ $\qquad$ 10% scary!

Prob Summary TOGETHER

A box contains 3 red, 4 green, and 5 white balls. One ball is picked at random. What is the probability that it is red or white?

What's different about this situation from the preceding ones? How will you use the formula:

$n(A \cup B) = nA + nB - n(A \cap B)$

$P(\text{not green}) = 1 - P(\text{green})$

$1 - \frac{4}{12} = \frac{8}{12}$

–or– disjoint!

$n(A \cup B) = nA + nB - \emptyset$

$3 + 5 = 8 \quad 8/12$

Chapter 5 Homework Problem 5

Two employees, Tom from Plant Operations and Becky from Campus Security, are supposed to check that the University Computing Center doors are locked after 6pm. On any given day, the probability that Tom will check the doors is 0.96 and the probability that Becky will check is 0.98. The probability that they both check is 0.95. What is the probability that neither of them will check the doors?

Prob Summary Problem - TOGETHER

The probability that a radioactive substance emits at least one particle during any given hour is 0.008. What is the probability that it does not emit any particle at all during any given hour?

$P(\text{none}) = 1 - P(\text{at least } 1) = 1 - .008 = .992$

Chapter 5 Homework Problem 6

Explain why it is not possible to have two events, A and B, with either of the following probabilities.

A. P(A) = 0.6 and P(A and B) = 0.8

B. P(A) = 0.7 and P(A or B) = 0.6

Prob Summary Problem TOGETHER

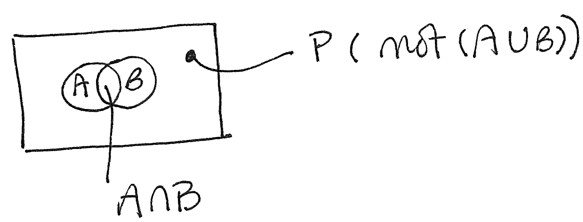
If A and B are events with 0.6 and 0.7 as their respective probabilities. Further P(A or B) = 0.9. Find P(not (A and B)).

Or: Union….And: Intersection

A .6

B .7

$A \cap B = .9$



$P(A \cup B) = .6 + .7 - .9 = .4$

$1 - .4 = .6$

Chapter 5 Homework Problem 7

Past records indicate that the probability that a new customer will open a checking account is 0.7 and the probability that a new customer will open a savings account is 0.4 while the probability of a new customer opening both is 0.25.

What is the probability that a new customer will open a savings account or a checking account?

What is the probability that the customer will open neither and opt for a different service entirely?

5.6 Geometric Probability

Geometric probability comes up in problems with spinners and with rectangular grids. The formula changes just slightly with this type of problem. Before we had

Probability: $\frac{\textit{favorable outcomes}}{\textit{total possible}}$

Now we'll have:

Geometric Probability: $\frac{\textit{area favorable outcomes}}{\textit{area total possible}}$

Look at the dart problem on page 106 oyo.

There's a spinner problem on page 142 – 143, that you can check out, too.

Be sure to read this carefully OYO.

Chapter 5 Summary

Problem 7, 11, 15 are very typical test problems

We have a 5 question popper and 3essays in the script. Turn in the essays in CourseWare under assignments. Homework in the script: 6 problems. Homework in the text:
Chapter 5 2, 6, 8, 10, 14